Describing the Strength of Visible Light

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INTRODUCTION

In many types of technical work it is necessary to describe the "strength"[1] of visible light. The matter is complicated by the fact that there are four distinct circumstances in which the strength of light is a consideration, each having its own dimensionality and unit of measure.

In this article we describe these four circumstances and the way in which the strength of light is described for each.

BACKGROUND

In the discussions of describing the strength of light, several crucial concepts are encountered with which the reader may not be familiar. In this section, we will give some insight into these crucial concepts.

Dimensionality

Dimensionality[2] is the property of a physical quantity that shows how it relates to the seven fundamental physical quantities: length, mass, time, electric current, thermodynamic temperature, amount of substance,[3] and luminous intensity. Any physical quantity can be described in terms of one or more of these seven quantities.

For example, the width of an object is a quantity with dimensionality *length*. The area of a geometric figure is a quantity with

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[1] I use the rather vague term "strength" here rather than, for example, "intensity" or "brightness", since all those terms have specific meanings applicable to only one of the four "circumstances" in which we may encounter the need for light measurement and description.

[2] Often called by mathematicians "dimension". As that word has a quite different common meaning, we will use the admittedly-more-clumsy term *dimensionality* here to avoid any misunderstanding.

[3] This is the property that relates to the notion, in chemistry, of "one mole" of a substance. It is related to the matter of molecular weight.

dimensionality *length squared*. The velocity of a moving object is a quantity with dimensionality *length per (unit)*[4] *time*.

We can write the dimensionality of a quantity in an algebraic form. For example, we can write the dimensionality of velocity, "length per (unit) time", this way: l/t, where l represents length, t represents time, and the word "per" in the verbal description is represented by the division sign ("/"). We can write the dimensionality of area, "length squared", as l^2. We can write the dimensionality of electric charge, "current-time"[5], this way: It, where I represents current and t represents time. Often we use a dot to represent multiplication, as for example $I \bullet t$, in order to avoid any misunderstanding that the adjacent symbols form a word.

Now consider the quantity "number of eggs in this box". It does not work in terms of any of the seven fundamental physical quantities. It works in terms of a just plain number (sometimes called a "counting number" to emphasize this). Such a quantity is said to be *dimensionless*.

The unit in which a quantity is described must have a dimensionality consistent with the dimensionality of the quantity. There are of course many different units for any given quantity. Length may be reckoned in units of inch, foot, yard, meter, fathom, furlong, or many others.

In modern engineering and scientific work, it is preferred to use only the units of the International System of Units (SI, from the first two initials of its name in French), the "modern metric system". Under the SI, length is always expressed in terms of the unit *meter* (or in a multiple or submultiple of it, such as *kilometer* or *nanometer*).

Units for derived quantities are formed from one or more of the basic units. These compound units can be written with the symbols for the units in the same algebraic form as that in which we represented dimensionality. In the case of units, we do not commonly use the dot to indicate multiplication, but rather a hyphen, in both verbal and symbolic forms: electric charge is measured in the SI unit *coulomb*, which corresponds to the ampere-second, or A-s.

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[4] The word "unit" is often included for clarity, but is not needed for the technical definition.. When we get into more complicated expressions of dimensionality, we often eliminate the word "unit".

[5] Meaning "current times time".

The quantity angle is an interesting case. Angle does not correspond to any of the seven fundamental physical quantities. In fact, angle can be thought of as working by counting “revolutions”. Thus angle is a dimensionless quantity.

Angle is nevertheless not unitless. Common units for measuring angle are the *degree* (1/360 of a revolution) and the *radian* ($1/2\pi$ revolution). Both are “dimensionless units”.

The unit *radian* is defined thus: if we construct two lines outward from the center of a circle of radius r, and the length of the circumference which they embrace is also r, then the angle between them is one radian.

Solid angle

If we look through the viewfinder of a camera, we are able to see a certain amount of space. We can describe it in terms of two angles. Perhaps the amount of space we can see is 30 degrees horizontally and 20 degrees vertically. But how do we describe the entire amount of space we can see (independent of its shape). The quantity we use is *solid angle*. If we have a cone or pyramid of infinite “length”, how do we describe “how big it is inside”? We do that in terms of its solid angle (the solid angle at its apex, to be precise).

Solid angle is measured in terms of the unit *steradian*.[6] If we construct a cone or pyramid from the center of a sphere having radius r, and the region of the sphere surface embraced by the cone or pyramid has an area r^2, then the solid angle of the apex of the cone or pyramid is one steradian.

RADIOMETRY AND PHOTOMETRY

Measurement and description of the “strength” of electromagnetic radiation (in several situations) constitutes the field of *radiometry*. Visible light is electromagnetic radiation, and thus the concepts of radiometry apply to it just as to other forms of electromagnetic radiation—at least when we are not interested in the strength of the light from the perspective of human perception.

When we are interested in the strength of light from the perspective of human perception, we enter the parallel field of *photometry*.

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[6] The particle *ster* comes from the Greek and means “solid” Recall that the term “stereophonic sound” was based on the concept that this was “solid sound”.

The quantities and units of photometry parallel those of radiometry, with the important difference that photometric concepts reflect the differing "sensitivity" of the human eye at different wavelengths. At any given wavelength, there is a standardized relationship between the corresponding radiometric and photometric quantities.

PHOTOMETRIC QUANTITIES AND UNITS

There are four different circumstances to which the general notion of the strength of light apply. Each has it own dimensionality and its own SI unit.

We will in many cases mention the most-common of the many non-SI units for the quantity.

Luminous flux

Luminous flux is the quantity that characterizes the total strength of a body of light. That body of light could be, for example, the total luminous output of a lamp, or the portion of that total luminous output that passes out through a nearby window, or the total amount of sunlight falling on a region of specified area.

The SI unit of luminous flux is the *lumen* (lm). The lumen is not one of the fundamental SI units. (It probably should have been, as it is at the root of the photometric food chain.)

Luminous flux is parallel to the concept of *power* in radiometry, and has the same dimensionality.

At any given wavelength, there is a known relationship between the luminous flux of a body of light (in lumens) and the power in the body of light (in watts). In fact, the modern definition of the lumen is (indirectly) based on that relationship at a wavelength of 555 nm, where 1 lumen is equivalent to 1/683 watt.[7]

A common non-SI unit of luminous flux is the *spherical candlepower*.[8]

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[7] This actually occurs through the definition of the *candela*, the unit we will encounter next.

[8] A hypothetical lamp exhibiting a *luminous intensity* (see the next section) of one candlepower in every direction is said to have a total output of one *spherical candlepower*.

Luminous intensity

The strength of light emission in a particular direction from an emitter of very small size (from the perspective of the viewer)—a "point source"—is the *luminous intensity* in that direction. Its dimensionality is *luminous flux per (unit) solid angle*. It is in effect the solid-angular density of luminous flux.

The SI unit of luminous intensity is the *candela* (cd). The candela corresponds to *one lumen per steradian* (lm/sr). The candela is one of the fundamental SI units.[9]

Luminous intensity is parallel to the concept of *radiation intensity* in radiometry.

It may at first seem peculiar that solid angle is involved in the concept if luminous intensity, Couldn't we just speak of the amount of luminous flux emitted in a certain direction? In fact, **no** flux flows along any particular line from the source[10], just as there is no material at a certain point within an object. To have flux, we must have something for it to flow through, a non-zero solid angle. (It can be as small as we wish to contemplate, but not of zero size.)

Thus the indicator of the strength of emission (in a particular direction) is in terms of the ratio of the luminous flux to the solid angle through which it flows, where the solid angle we think of is arbitrarily small but centered on the direction of interest.

Note that the concept of *luminous intensity* does not involve distance from the emitter. It describes the emission, not the effect the emission produces at some distant point. (It of course influences the effect produced at a distant point, as we'll see shortly under *illuminance*.)

A common non-SI unit of luminous intensity is the *candlepower* (sometimes called *beam candlepower* to distinguish it from *spherical candlepower* used as a unit of luminous flux).

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[9] It got this honor, rather than the *lumen*, since at the time the SI was being formulated there were practical ways to measure luminous intensity but not luminous flux.

[10] Recall that a line is infinitely thin, and as such does cannot constitute a "conduit" for luminous flux any more than an infinitely-thin pipe could convey water.

Luminance

Luminance tells us the "brightness" of the light emission from a source whose size (from the perspective of the viewer) is not insignificant—an "extended source". Its dimensionality is *luminous intensity per unit area*, or *luminous flux per unit solid angle per unit area*.

In terms of the SI, the unit of luminance is the *candela per square meter*[11] (cd/m^2) It can also be expressed as the *lumen per steradian per square meter* ($lm/sr\text{-}m^2$).

We can perhaps best understand the concept and dimensionality of *luminance* by imagining that the emitting surface is populated with a very large number of point sources, each emitting with a certain *luminous intensity* (in terms of *lumens per steradian*). The brightness of the surface, as seen by a human observer, is proportional to the luminous intensity of these point sources and also to how tightly-packed they are—how many of them there are per square meter of the surface.

In radiometry we are rarely concerned with emission from an extended source, and so the parallel concept to luminance is not often encountered.

Photographic light meters of the "reflected light" type measure *luminance*.

A common non-SI unit of luminance is the *foot-lambert*.

Luminance is often called, outside the SI, "brightness".

Illuminance

Illuminance tells us the "strength" of light falling on a surface.[12] Its dimensionality is *luminous flux per unit area*.

The SI unit of illuminance is the *lux*, which corresponds to *one lumen per square meter*.

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[11] This compound unit does have its own name, the *nit*, but that name is not actually a part of the SI and is not too widely used.

[12] Or light crossing some imaginary plane in space.

Illuminance is parallel to the concept of *power flux density* (PFD) in radiometry.

The illuminance created at a distant location from the emission from a point source is proportional to the luminous intensity of the source in the direction toward the location and inversely proportional to the square of the distance from the source to the location (the "inverse square law").

The *luminance* (brightness) of an object, in most cases of interest, is proportional to the total *illuminance* it receives from light sources and also to its *reflectance*. (Information on reflection is found in Appendix A.)

Photographic light meters of the "incident light" type measure *illuminance*.

Illuminance is sometimes called, outside the SI, "illumination".

A common non-SI unit of illuminance is the *footcandle*.

Light quantity

Not strictly speaking, one of the four concepts of the "strength" of light, but important nevertheless is "light quantity", which may reasonably be thought of as "photometric energy". It is the photometric parallel to *energy* in radiometry. Its dimensionality is *luminous flux times time*. Its SI unit is the *lumen-second* (lm-s).

The related quantity *luminous flux times time per unit area* is the quantity which influences the degree of exposure of photographic film. As such, it is often called "exposure" (although, unfortunately, that term is used with another meaning as well in the field of photography.[13] Its SI unit is the *lumen-second per square meter* (lm-s/m^2).

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[13] That other meaning is the combination of exposure time (shutter speed) and relative aperture.

APPENDIX A

Reflection

Introduction

Most objects we observe do not emit light but rather reflect light from another source, such as the sun or a lamp.

Many situations in which we are concerned with describing the strength of light relate to the effects of reflection. Here we will spend a little time talking about reflection.

Kinds of reflection

Surfaces may exhibit two kinds of reflection. *Specular reflection* is the reflection afforded by a mirror. A light ray striking a specular reflecting surface is reflected as a ray. The angle at which it arrives and the angle at which it leaves (both measured with respect to a line from the point of "impact" and perpendicular to the surface) have the same magnitude but are opposite in direction.

Diffuse reflection is the reflection afforded by most surfaces we encounter. In diffuse reflection, when light strikes a surface, the reflected light departs in every direction on the same side of the plane (but not with uniform intensity—we'll get to that in a moment).

Many real surfaces exhibit "mixed" reflection, a combination of the specular and diffuse types. A piece of shiny metal, or a refrigerator with a glossy finish, are of this category.

An ideal diffuse surface obeys Lambert's law. Two of its important properties are:

- The distribution of radiation intensity of the reflected light is not affected by the angle from which the incident illumination comes,
- If we consider any tiny area of the surface (which we can treat as a point source), the light intensity in any direction is proportional to the cosine of the angle between that direction and a line from the point, perpendicular to the surface.

In view of the second of these, we might expect that the brightness of an illuminated "Lambertian" surface would vary with the angle from which we view the surface. However it doesn't—it is the same for any angle of view.

The reason has to do with the concept of *projected area*. If we observe a certain region on a surface not "head on" but rather from an angle, the area of the region will appear smaller. The decrease in apparent area goes as the cosine of the angle, measured from "head on", at which we view the surface.

Now let us consider our illuminated surface under the model discussed above in connection with luminance: the surface involves an enormous number of tiny "point sources", each emitting with the same luminous intensity. The brightness of the surface is the product of that luminous intensity and the area density of these point sources (the number per unit area).

Now, for our Lambertian surface, as we observe it from an increasing angle, the luminous intensity for each point source decreases as the cosine of the angle of view. But the area density of the point sources increases, since we see the same number of sources in what we see as a smaller area; the increase is **inversely** proportional to the cosine of the angle of view. These two effects cancel out, and thus the luminance (brightness) is the same from any direction of view.

A Lambertian surface will not necessarily reflect all the luminous flux incident on it, and it will not necessarily reflect the same fraction at all wavelengths.[14] The fraction of light reflected by a Lambertian surface, taking the differing response of the eye at different wavelengths into account, is called the *reflectance* of the surface[15] and is represented by the lower-case Greek letter "rho" (ρ).

If we illuminate a Lambertian surface of reflectance ρ with light having illuminance E (in lux), the reflected light will have luminance (brightness) L (in cd/m^2), as follows:

$$L = \frac{1}{\pi}\rho E$$

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[14] Gray paint, for example, reflects far less than all the light incident on it, and red paint does not reflect the same fraction of the light for all wavelengths.

[15] Also called its *albedo*, especially if we are speaking of astronomical objects, such as planets and their satellites.

The pi (π) gets into the deal because of the involvement of the unit of solid angle, the steradian, in the chain and the fact that the number of steradians in the solid angle embracing "all directions on the same side" of a surface is 2π.

If we do this for the most common non-SI units, the *foot candle* for illuminance (illumination), and the *foot lambert* for luminance (brightness), the relationship becomes:

$$L = \rho E$$

Why? Because the definition of the foot-lambert has the $1/\pi$ built in. Why? To make this important equation simpler!

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